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Alternative Approaches to Estimating VAR for Hedge Fund Portfolios

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INTRODUCTION

This chapter compares four different approaches to estimating value-at-risk (VAR) for hedge fund portfolios. We focus on approaches that are based on the thin-tailed normal, fat-tailed generalised error distributions (GEDs), an extreme value approach that approximates the tails of the return distribution asymptotically, and the Cornish-Fisher (CF) expansion that takes into account skewness and kurtosis of the empirical distribution. The main difficulty common to all these VAR models is the estimation of the required quantile of the loss distribution, since there is no analytical representation of this distribution. The VAR numbers calculated by a specific methodology are compared to the actual losses. The results indicate that accuracy is rather poor for VAR methods relying only on the first two moments of the loss distribution. The inclusion of higher moments through the CF expansion results in more accurate estimates of the actual VAR thresholds.

Hedge fund strategies vary substantially. They have great flexibility regarding asset classes, trading styles, markets, level of transparency and liquidity. They hold long and short positions, use leverage through borrowing, exploit arbitrage opportunities and

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trade in derivatives extensively. There are more than 10 distinct investment strategies used by hedge funds, each offering different risk–return characteristics. Extent of risk would be different for different hedge fund strategies due to their style of operation and selection of assets and markets. While there is now increasing evidence that hedge funds may offer relatively higher means and lower variances, such funds also give investors third- and fourth-moment attributes that are exactly the opposite to those that are desirable. Hedge fund returns come with negative skewness and higher kurtosis, which in turn causes non-normality in their return distributions. Especially after the collapse of Long-Term Capital Management (LTCM) in August 1998, the importance of sound risk management and measuring the performance on a risk-adjusted basis have been strongly emphasised for hedge funds.

Also recently, there has been an increased focus on the determination of capital requirements for hedge funds to meet catastrophic market risk. This increased focus has led to the development of various risk-measurement techniques for hedge fund portfolios. The primary technique is VAR, which determines the maximum expected loss on a portfolio of assets over a certain holding period at a given confidence level (probability).[1] Since hedge funds trade in multiple asset classes, a uniform measure of risk is needed. VAR does just that, for any traded instruments. VAR has gained increasing acceptance among many hedge funds in the past few years, and nowadays most hedge funds and funds of funds are using VAR to measure the risk to their portfolios. One advantage of VAR is that it is an intuitively appealing measure of overall risk that can be easily conveyed to senior management and investors.

This increased focus on the risk management of hedge funds also motivated academic and practitioner-oriented research in the area of hedge fund risk exposure. Jorion (2000) studies LTCM's VAR and estimates the amount of capital that is necessary to support its risk profile. He finds that LTCM severely underestimates its risk due to its reliance on short-term history and risk concentration. Agarwal and Naik (2002) use a mean-conditional VAR framework, and demonstrate the extent to which the mean-variance framework underestimates the tail risk. They show how the conditional VAR framework that explicitly accounts for the negative tail risk can be applied to construct portfolios of hedge funds. Gupta and

Liang (2001) use the extreme value theory (EVT) to estimate VAR thresholds, and infer capital adequacy from these VARs for the hedge fund industry. They also use the normal distribution to test the empirical performance of the extreme value approach.[2]

Theory and practice show that standard assumptions – either the investors have quadratic utility or the asset returns are jointly normally distributed – that justify the use of mean-variance theory cannot be applied to a risk-averse agent investing part of its wealth in a portfolio of hedge funds. Hedge fund instruments usually have significant negative skewness along with high kurtosis, which make their returns' distribution far from normal. Geman and Kharoubi (2003) support this hypothesis, and find that the assumption of normality is inappropriate for hedge funds, unlike typical stock and bond benchmarks. Working in a mean-variance framework implies that the investor is missing a significant part of the risk that is inherent in hedge funds.

Based on this conclusion, we define a VAR model that takes into account the mean, variance, skewness and kurtosis of the hedge fund returns' distribution. We define the risk of the portfolio using the standard VAR corrected by the CF (1937) expansion. We also use an extreme value approach of Bali (2001) that approximates the tails of the return distribution asymptotically instead of imposing a symmetric thin-tailed functional form like the normal or lognormal distribution. Although VAR measures based on the traditional parametric approach with normal density may provide acceptable estimates of the maximum likely loss under normal market conditions, Longin (2000), McNeil and Frey (2000) and Bali (2001) show that they fail to account for extremely volatile periods corresponding to financial crises. The previous literature on EVT indicates that the VAR measures based on the distribution of extreme returns, instead of the distribution of all returns, provide good predictions of catastrophic market risks during extraordinary periods.

We compare four alternative approaches to calculating VAR for hedge fund indices. We focus on approaches that are based on the thin-tailed normal distribution, the fat-tailed GED, an extreme value approach that approximates the tails of the return distribution asymptotically, and the CF expansion that takes into account skewness and kurtosis of the empirical distribution. The main difficulty common to all these VAR models is the estimation of the required quantile of the loss distribution, since there is no analytical representation of this distribution. The VAR numbers calculated by a specific methodology are compared to the actual losses using alternative performance measures: Determination Coefficient (or regression R^2), Theil Inequality Coefficient (TIC), heteroskedasticity-adjusted mean absolute (HMAE) and root mean square errors (HRMSE). The results indicate that the accuracy of VAR methods that rely only on the first two moments of the loss distribution is rather poor. The inclusion of higher moments through the CF expansion and the modelling of extreme tails of the return distribution, yield more accurate estimates of the actual VAR thresholds.

The chapter is organised as follows. The next section presents alternative VAR models. The following section, "Data", describes the hedge fund index returns data. The section "Empirical results" provides the empirical result and the following section concludes the chapter.

VALUE-AT-RISK MODELS

The traditional VAR models assume that the probability distribution of log-price changes (log returns) is normal. However, the distributions of log returns of financial assets are usually skewed to the left, have fat tails, and are peaked around the mode. The fat tails suggest that extreme outcomes happen much more frequently than would be predicted by the normal distribution.[3] This section presents alternative VAR models based on the normal distribution, the GED, the EVT and the CF expansion.[4]

In continuous time diffusion models, (log)-stock price movements are described by the following stochastic differential equation,

$$\mathrm{d}\ln P_t = \mu \mathrm{d}t + \sigma \mathrm{d}W_t \tag{1}$$

where W_t is a standard Wiener process with zero mean and variance of $\mathrm{d}t$, μ and σ are the constant drift and diffusion parameters of the geometric Brownian motion. In discrete time, equation (1) yields a return process:

$$\ln P_{t+\Delta} - \ln P_t = R_t = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t} \tag{2}$$

where Δt is the length of time interval in which the discrete time data are recorded and $\Delta W_t = \varepsilon\sqrt{\Delta t}$ is the Wiener process with zero mean and variance of Δt since ε is a random variable drawn from the standard normal density, ie, $E(\varepsilon) = 0$ and $E(\varepsilon^2) = 1$. Equation (2) implies that assuming $\Delta t = 1$, $\varepsilon = (R - \mu)/\sigma$.

The critical step in calculating VAR measures is the estimation of the threshold point defining what variation in returns R_t is considered to be extreme. Let α be the probability that R_t is less than the threshold $\Im$. That is,

$$\Pr\left(R_t < \Im\right) = \Pr\left(\varepsilon < a = \frac{\Im - \mu}{\sigma}\right) = \alpha \qquad \textbf{(3)}$$

where $\Pr(\cdot)$ is the underlying probability distribution, μ and σ are the mean and standard deviation of R_t. In the traditional parametric VAR model with normal distribution, $\alpha = 1\%$ and $a = -2.326$:[5]

$$\Im_{\text{Normal}} = \mu - 2.326\sigma \qquad \textbf{(4)}$$

There is substantial empirical evidence that the distribution of hedge fund returns is typically skewed to the left and leptokurtic. That is, the unconditional return distribution shows high peaks, fat tails and more outliers on the left tail. This implies that extreme events are much more likely to occur in practice than would be predicted by the thin-tailed normal distribution. This also suggests that the normality assumption can produce VAR numbers that are inappropriate measures of the true risk faced by hedge funds. To account for excess kurtosis in the data, we use the fat-tailed GED that accounts for the non-normality of returns and relatively infrequent events. The GED density used to model VAR and observed leptokurtosis in hedge fund data is given by equation (5):

$$f_v(\varepsilon_t) = \frac{v \ \exp[(-1/2)\left|\varepsilon_t/\Pi\right|^v]}{\Pi \ 2^{[(v+1)/v]} \ \Gamma(1/v)} \qquad \textbf{(5)}$$

where $\varepsilon_t = (R_t - \mu)/\sigma$, $\Gamma(\cdot)$ is the gamma function,

$$\Pi = \left[\frac{2^{(-2/v)}\Gamma(1/v)}{\Gamma(3/v)}\right]^{1/2},$$

and $v > 0$ is the degrees of freedom or tail-thickness parameter. For $v = 2$, the GED yields the normal distribution, while for $v = 1$ it yields the Laplace or the double exponential distribution. If $v < 2$, the density has thicker tails than the normal, whereas for $v > 2$ it has thinner tails.

VAR is simply a specific percentile of a portfolio's potential loss distribution over a holding period. Assuming $R_t \sim f_v(\varepsilon_t)$, where $f_v(\varepsilon_t)$ is the GED density in equation (5), VAR is the solution to:

$$\int_{-\infty}^{\Im_{GED}(\alpha)} f_v(\varepsilon)d\varepsilon = \alpha \tag{6}$$

where $\Im_{GED}(\alpha)$ is the VAR threshold based on the GED density with a loss probability of α. Equation (6) indicates that VAR can be calculated by integrating the area under the probability density function of the GED given by equation (5).

Since hedge fund index returns are skewed and fat-tailed we cannot use a VAR formula that assumes a normal distribution. An alternative method is to use the moments of the distribution. We estimate VAR using the CF (1937) expansion that adjusts the traditional VAR with the skewness and kurtosis of the empirical distribution:

$$\Im_{CF}(\alpha) = \mu - \Omega(\alpha) \times \sigma \tag{7}$$

where μ is the average return, σ is the standard deviation and $\Omega(\alpha)$ is the critical value based on the loss probability level, skewness and kurtosis of the empirical distribution:

$$\begin{aligned}\Omega(\alpha) = z(\alpha) + \frac{1}{6}(z(\alpha)^2 - 1)S + \frac{1}{24}(z(\alpha)^3 - 3z(\alpha))K \\ - \frac{1}{36}(2z(\alpha)^3 - 5z(\alpha))S^2\end{aligned} \tag{8}$$

where $z(\alpha)$ is the critical value from the normal distribution for probability $(1 - \alpha)$, S is the skewness, and K is the excess kurtosis.[6] Equation (7) indicates that the CF expansion allows us to compute VAR for a distribution with asymmetry and leptokurtosis. Note that, if the distribution is normal, S and K are equal to zero, which makes $\Omega(\alpha)$ equal to $z(\alpha)$.

We should also note that the symmetric GED density does not allow for asymmetric treatment of upside potential and downside risk. In other words, the GED does not take into account skewness although it accounts for leptokurtosis. In practice, we know that portfolios respond asymmetrically to good and bad states. An important advantage of the CF (1937) expansion is that it allows investors to treat losses and gains asymmetrically. In addition, as presented in Table 1, skewness statistics of hedge fund returns are found to be significant at the 1% or 5% level. Therefore, we expect that the VAR thresholds obtained from the CF (1937) expansion, $\Im_{CF}(\alpha)$, be more accurate than the thresholds estimated by the GED and normal density, $\Im_{GED}(\alpha)$ and $\Im_{Normal}(\alpha)$.

An alternative provided in the EVT literature is to work with the extreme value distribution $H(\Im)$ instead of $Pr(\cdot)$, and then determine the threshold level $\Im$ by going backwards from α to $\Im$ by solving:

$$H(\Im) = 1 - \alpha \tag{9}$$

given the value of α. As shown in Bali (2001), the generalised Pareto distribution (GPD)

$$GPD_{min}(M;\mu,\sigma,\xi) = \left[1 + \xi \left(\frac{\mu - M}{\sigma}\right)\right]^{-1/\xi} \tag{10}$$

yields the following VAR threshold:

$$\Im_{EVT} = \mu + \left(\frac{\sigma}{\xi}\right)\left[\left(\frac{\alpha n}{k}\right)^{-\xi} - 1\right] \tag{11}$$

where M denotes the minimal returns, k and n are the number of extremes and the number of total data points, respectively. Once the location (μ), scale (σ) and shape (ξ) parameters of the GPD are estimated one can find the VAR threshold, $\Im_{EVT}$, based on the choice of confidence level α.[7] As discussed in Panel 1, the regression method, which is based on the order statistics of extremes from a uniform parent, is used to estimate the tails of the hedge fund returns distribution.[8] Note that the location and scale parameters of the GPD approximately correspond to the mean and standard deviation parameters of the normal distribution.

DATA

The hedge fund indices used in this study were obtained from Hedge Fund Research, Inc. (HFR), which is one of the best-known and largest hedge fund databases currently available. There were several reasons behind choosing HFR indices. First, they maintain the

Table 1 Hedge fund index return data: January 1990–June 2002 ($n = 150$)

HFRI index	Mean (%)	Standard deviation (%)	Skewness	Kurtosis	JB	Worst (%)	Best (%)
Convertible arbitrage	0.90	1.02	−1.31***	6.44***	79.35***	−3.19	3.33
Distressed securities	0.85	1.63	−1.81***	12.36***	428.82***	−8.50	5.06
Emerging markets (total)	0.63	4.64	−0.75***	6.75***	69.26***	−21.02	14.80
Equity hedge	1.31	2.83	0.30	4.38***	9.45***	−7.65	10.88
Equity-market-neutral	0.80	0.97	0.03	3.30	0.41	−1.67	3.59
Equity non-hedge	1.16	4.33	−0.47***	3.45*	4.62*	−13.34	10.74
Event-driven	1.09	1.93	−1.34***	8.75***	171.05***	−8.90	5.13
Fixed income (total)	0.70	0.96	−1.26***	7.09***	97.95***	−3.27	3.28
Fixed income: arbitrage index	0.53	1.29	−2.77***	15.29***	772.26***	−6.45	3.04
Fixed income: high-yield index	0.54	1.42	−1.90***	10.54***	303.39***	−7.16	2.98
Fund-of-funds index	0.63	1.89	−0.22	6.22***	44.88***	−7.47	6.85
Macro index	0.84	2.28	0.02	3.48**	1.02	−6.40	6.82
Market-timing index	0.99	2.11	0.18	2.35	2.38	−3.28	5.96
Merger-arbitrage index	0.94	1.10	−2.66***	15.18***	750.19***	−5.69	2.47
Relative-value arbitrage index	0.87	1.00	−2.84***	20.47***	1,434.12***	−5.80	2.80
Short-selling index	0.52	7.04	0.16	3.95***	4.29	−21.21	22.84
Statistical arbitrage index	0.73	1.10	−0.17	3.04	0.51	−2.00	3.60

Standard errors of the skewness and kurtosis estimates, computed under the null hypothesis that hedge fund returns are normally distributed, are $\sqrt{6/n}$ and $\sqrt{24/n}$, respectively. Jarque-Bera, $JB = n[(S^2/6) + (K-3)^2/24]$, is a formal test statistic for testing whether the returns are normally distributed, where n denotes the number of observations, S is skewness and K is kurtosis. The test statistic distributed as the Chi-square with two degrees of freedom measures the difference of the skewness and kurtosis of the series with those from the normal distribution. The critical values with two degrees of freedom at the 10%, 5%, and 1% level of significance are $\chi^2_{(2,0.10)} = 4.61$, $\chi^2_{(2,0.05)} = 5.99$, and $\chi^2_{(2,0.01)} = 9.21$.
*, **, *** denote the 10%, 5% and 1% level of significance, respectively.

PANEL 1 THE GENERALISED PARETO DISTRIBUTION

Extreme movements are measured by the monthly changes in hedge fund returns R. Let us call f(R) the probability density function (PDF) and F(R) the cumulative distribution function (CDF) of R, which can take values between l and u.[9] Extremes are defined as the maxima and minima of the n independent and identically distributed random variables $R_1, R_2, \ldots, R_n$. The asymptotic distribution of extremes can be estimated based on the concept of GPD. Excesses over either high or low thresholds can be modelled by the GPD. First, we choose a low threshold l so that all $R_i < l < 0$ are defined to be in the negative tail of the distribution, where $R_1, R_2, \ldots, R_n$ are a sequence of hedge fund returns on months $1, 2, \ldots, n$. Then we denote the number of exceedances of l (or returns lower than l) by

$$N_u = \text{card}\{i: i = 1, \ldots, n, R_i > n\}, \qquad \textbf{(I)}$$

and the corresponding excesses by $M_1, M_2, \ldots, M_{N_u}$. The excess distribution function of R is given by:

$$F_l(y) = P(R - l \geq y | R < l) = P(M \geq y | R < l), \quad y \leq 0 \qquad \textbf{(II)}$$

Using the threshold l, we now define the probabilities associated with R:

$$P(R \leq l) = F(l) \qquad \textbf{(III)}$$

$$P(R \leq l + y) = F(l + y) \qquad \textbf{(IV)}$$

where $y < 0$ is an exceedance of the threshold l. Finally, let $F_l(y)$ be given by

$$F_l(y) = \frac{F(l) - F(l+y)}{F(l)} \qquad \textbf{(V)}$$

We thus obtain the $F_l(y)$, the conditional distribution of *how* extreme a R_i is, given that it already qualifies as an extreme. Pickands (1975) shows that $F_l(y)$ will be very close to the GPD in equation (10) if l is a low threshold:

$$\text{GPD}(M; \mu, \sigma, \xi) = \left[1 + \xi \left(\frac{\mu - M}{\sigma}\right)\right]^{-1/\xi}. \qquad \textbf{(VI)}$$

where μ, σ, and ξ are the location, scale, and shape parameters of the GPD, respectively. The *shape* parameter ξ, called the tail index, reflects the fatness of the distribution (ie, the weight of the tails), whereas the parameters of *scale* σ and of *location* μ represent the dispersion and average of the extremes, respectively.

Estimation procedure

The regression method, which is based on the order statistics of extremes from a uniform parent, is used to estimate the parameters of GPD. As described in Gumbel (1958), the sequence of minimal values $X_{\min,1}, X_{\min,2}, \ldots, X_{\min,n}$ is arranged in increasing order to get an order statistic $\tilde{X}_{\min,1}, \tilde{X}_{\min,2}, \ldots, \tilde{X}_{\min,n}$, which satisfies $\tilde{X}_{\min,1} \le \tilde{X}_{\min,2} \le \ldots \tilde{X}_{\min,n}$. The order statistics of a sample of size n, coming from a *uniform* U(0,1) parent population are used in the regression method to estimate the parameters of the extreme value distribution. The importance of this population is based on the fact that any other continuous population can be easily transformed to the uniform and some properties of these order statistics can be directly translated to properties of the order statistics of a sample drawn from initial population.

The r-th order statistic has the following PDF:

$$f_{X_{r:n}}(x) = \frac{x^{r-1}(1-x)^{n-r}}{B(r, n-r+1)}; \quad 0 \le x \le 1, \tag{VII}$$

where $B(r, n - r + 1)$ is the beta function. The ordered maxima, $\tilde{X}_{\min,r}$, have the following mean:

$$\mu_{r:n} = \int_0^1 \frac{x^r(1-x)^{n-r}\,dx}{B(r, n-r+1)} = \frac{r}{n+1}. \tag{VIII}$$

For each r ranging from 1 to n, the frequency $GPD_{\min,n}(\tilde{X}_{\min,r})$ is a random variable lying between 0 and 1. The mean of the r-th frequency, $\mu_{r:n} = E[GPD_{\min,n}(\tilde{X}_{\min,r})]$, is equal to $r/(n + 1)$. The regression method equates the ordered extreme observation $GPD_{\min,n}(\tilde{X}_{\min,r})$ to its theoretical mean, $r/(n + 1)$:

$$E\left[GPD_{\min,n}(\tilde{X}_{\min,r})\right] = E\left[\left(1 + \xi_{\min}\left(\frac{\mu_{\min} - \tilde{X}_{\min,r}}{\sigma_{\min}}\right)\right)^{-1/\xi_{\min}}\right] = \frac{r}{n+1} \tag{IX}$$

which can be reduced to an empirically estimable form by taking the logarithm of both sides and adding an error term $\eta_{\min}$:

$$-\ln\left(\frac{r}{n+1}\right) = -\frac{1}{\xi_{\min}}\ln\sigma_{\min} + \frac{1}{\xi_{\min}}\ln(\sigma_{\min} + \xi_{\min}(\mu_{\min} - \tilde{X}_{\min,r})) + \eta_{\min} \tag{X}$$

Consistent parameter estimates of the GPD are obtained by minimising the sum of squared residuals from the nonlinear regression equation given in (X).

performance of the liquidated funds, therefore mitigating notoriously known survivorship bias.[10] Second, HFR indices have a critically large number of constituents, which gives a better representation for return distribution of the hedge fund strategies. Finally, HFR indices are equally weighted, which mitigates concentration in a few hedge funds with large assets under management. Our data contain monthly returns from January 1990 to June 2002 for 17 hedge fund strategy indices, which reflect the monthly net of fee returns. Strategies of these indices are described briefly in Panel 2.

PANEL 2 HEDGE FUND STRATEGY DESCRIPTIONS

- *Convertible arbitrage* hedge funds focus on the mispricing of convertible bonds. A typical position involves a long position in the convertible bond and a short position in the underlying asset.
- *Distressed securities* hedge funds take long positions on the equity or debt of companies that are in or facing bankruptcy.
- *Emerging-markets* hedge funds invest in equity and fixed-income securities of emerging markets around the world.
- *Equity-hedge* hedge funds represent the original hedge fund model of Alfred Winslow Jones. They invest in equities and take long and short positions.
- *Equity-market-neutral* hedge funds simultaneously take long and short positions of the same size within the same market to maintain zero market risk.
- *Equity non-hedge* hedge funds take long positions in equities.
- *Event-driven* hedge funds take positions on corporate events such as a merger, an acquisition or a bankruptcy.
- *Fixed-income arbitrage* hedge funds tend to profit from price anomalies between related fixed-income securities.
- *Funds of funds* are funds that invest in a pool of hedge funds. Funds of funds could be diversified, allocating capital to the various hedge funds strategies; or niche, investing only specific hedge fund strategy.
- *Macro* funds rely on macroeconomic analysis to take bets on major risk factors, such as currencies, equities, interest rates and commodities.
- *Market-timing* hedge funds use technical and fundamental models to identify turning points in the markets and take long and short positions based on their view.
- *Merger-arbitrage* hedge funds seize the opportunity to invest after merger event has been announced. They take long position on the

shares of the target company, and short position on the shares of the acquiring company.

- *Relative-value arbitrage* index covers convertible arbitrage, fixed-income arbitrage, statistical arbitrage and market-neutral strategies.
- *Short-selling* hedge funds are essentially equity hedge funds that maintain a consistent net short exposure.
- *Statistical arbitrage* hedge funds use market-neutral strategy by taking long and short positions in related securities. These types of hedge fund use quantitative models to determine price anomalies between cointegrated securities.

Table 1 presents descriptive statistics for 17 hedge fund indices.[11] More specifically, the table reports the means, standard deviations, skewness, kurtosis, Jarque-Bera statistics, the worst and the best month for each index. The table shows that there is substantial variation between different strategies. Among all indices, short-selling index has the lowest average return, 0.50% per month, with highest standard deviation, 6.61%. Equity hedge index provides the highest average monthly return, 1.52%, with fourth highest volatility, 2.68%. Twelve indices display negative skewness, and 13 indices display significantly higher kurtosis than that of the normal distribution. This means that large negative returns on hedge fund indices are much more likely to occur than would be predicted by the normal distribution. The Jarque-Bera statistics indicate that most hedge fund index returns are not normally distributed. These results signal that the risk profile of hedge funds cannot be accurately described by the mean and standard deviation alone. Investors will also have to measure the distribution's skewness, kurtosis and maybe higher moments to assess the risk of their investments more accurately. These results also indicate that, when hedge funds are involved, calculating VAR with the assumption of normality may result in misleading estimates of actual thresholds.

EMPIRICAL RESULTS

The VAR measures for the normal distribution, $\Im_{\text{Normal}}$, and CF expansion, $\Im_{\text{CF}}$, are obtained using equations (4) and (7), which depend on the mean and standard deviation of returns as well as the critical value related to a given confidence level (or loss probability level).[12,13] As shown in equation (6), the VAR threshold for the GED is calculated by integrating the area under the PDF. The first step for computing $\Im_{\text{GED}}$ is to estimate the mean (μ), standard deviation (σ), and tail-thickness (v) parameters of the GED density. The parameters of the GED given in equation (5) are estimated by maximising the log-likelihood function:

$$\text{Log}\,\text{L}_{\text{GED}} = \ln(v/2) + 0.5\ln\Gamma(3/v) - 1.5\ln\Gamma(1/v) - 0.5\sum_{t=1}^{n}\ln\sigma^2 - \exp\left[(v/2)\,(\ln\Gamma(3/v) - \ln\Gamma(1/v))\right] \times \sum_{t=1}^{n}\left|\frac{R_t - \mu}{\sigma}\right|^{v} \quad \textbf{(12)}$$

Panel 3 shows the maximum likelihood estimates of the GED for the hedge fund index returns. As discussed earlier, for the tail thickness parameter $v = 2$, the GED density equals the standard normal density. However, the estimates of v turn out to be highly significant and less than 2 for all hedge fund indices except for the market-timing index. The tail-thickness parameter is estimated to be 3.21 and significant at the 1% level, which implies that the distribution of returns on the market-timing index has thinner tails than the normal distribution. The degrees-of-freedom parameter, v, for the other 16 hedge fund return indices is estimated to be in the range of 0.86 to 1.92. Although not presented in the paper, comparing the maximised log-likelihood functions of the models estimated with the GED and normal distributions indicates that v is statistically different from 2, implying that the distribution of hedge fund returns is much more leptokurtic than the corresponding normal distribution.

As shown in equation (11), the VAR threshold for the extreme value approach, $\Im_{\text{EVT}}$, depends on the estimated location (μ), scale (σ), shape (ξ) parameters of the GDP, and the loss probability level (α). Panel 4 presents the regression method estimates of the GPD for the hedge fund index returns.[14] We should note that the extremes (or minimal returns) used in the regressions are obtained from the original monthly data described in Table 1. Following the EVT for the GPD, we define the extremes as excesses over high thresholds (see Embrechts, Kluppelberg and Mikosch, 1997, pp. 352–5). Specifically, the extreme monthly returns correspond to the 10% left tail of the empirical distribution.[15] As shown in Panel 4,

PANEL 3 MAXIMUM LIKELIHOOD ESTIMATES OF THE GED

This table displays the maximum likelihood estimates of the GED for the hedge fund index returns. The parameter estimates of the fat-tailed GED density with asymptotic *t*-statistics are shown in parentheses for each index. The maximised log-likelihood values (Log-L) are reported in the last column.

HFRI index	μ	σ	v	Log-L
Convertible arbitrage	0.0110	0.00009	1.0613	493.57
	(18.351)	(6.4774)	(7.1013)	
Distressed securities	0.0115	0.00032	1.0320	398.05
	(10.639)	(6.5554)	(7.7130)	
Emerging markets (total)	0.0154	0.00206	1.2673	253.42
	(4.6304)	(7.1836)	(8.1445)	
Equity hedge	0.0155	0.00071	1.5559	330.22
	(7.3635)	(7.9537)	(6.6601)	
Equity-market-neutral	0.0085	0.00009	1.5872	484.62
	(11.233)	(6.8992)	(4.9989)	
Equity non-hedge	0.0153	0.00178	1.7355	260.40
	(4.2219)	(8.0361)	(5.9096)	
Event-driven	0.0139	0.00034	1.1250	391.43
	(11.381)	(6.8396)	(8.5632)	
Fixed income (total)	0.0103	0.00011	0.9347	484.20
	(19.411)	(5.8557)	(7.1842)	
Fixed income: arbitrage index	0.0068	0.00017	0.8993	451.44
	(10.473)	(6.0420)	(8.1358)	
Fixed income: high-yield index	0.0084	0.00034	0.8572	400.84
	(11.230)	(5.7321)	(7.2413)	
Fund-of-funds index	0.0084	0.00029	1.1389	401.30
	(7.4113)	(6.7447)	(7.4988)	
Macro Index	0.0132	0.00065	1.7175	335.24
	(6.2713)	(7.5233)	(4.6762)	
Market-timing index	0.0113	0.00041	3.2121	372.49
	(6.9715)	(10.405)	(3.6765)	
Merger-arbitrage index	0.0123	0.00012	0.8741	479.39
	(24.112)	(6.1758)	(8.7394)	
Relative-value arbitrage index	0.0108	0.00011	1.1435	472.80
	(15.321)	(6.9415)	(10.042)	
Short-selling index	0.0029	0.00434	1.4604	195.55
	(3.7560)	(7.4301)	(6.1011)	
Statistical arbitrage index	0.0083	0.00012	1.9173	458.26
	(9.0337)	(8.6775)	(5.5895)	

PANEL 4 REGRESSION METHOD ESTIMATES OF THE GPD

This table displays the regression method estimates of the GPD for the hedge fund index returns. The extremes (or minimal returns) used in the regressions are obtained from the 10% left tail of the empirical distribution. The parameter estimates of GPD with asymptotic *t*-statistics are shown in parentheses. The estimated VAR thresholds for the extreme value approach ($\Im_{EVT}$) are calculated based on μ, σ, ξ, and $\alpha = 0.01$.

HFRI index	μ	σ	ξ	$\Im_{EVT}$
Convertible arbitrage	−0.001241 (−1.6094)	0.012181 (7.5701)	−0.036568 (−0.3703)	−0.0281
Distressed securities	−0.005771 (−7.9773)	0.009458 (6.8114)	0.658291 (5.9661)	−0.0568
Emerging markets (total)	−0.045606 (−39.871)	0.013918 (6.8220)	0.834502 (7.7112)	−0.1428
Equity hedge	−0.019301 (−35.130)	0.007216 (6.9112)	0.633852 (5.8606)	−0.0569
Equity-market-neutral	0.001129 (0.9311)	0.018314 (6.2403)	−0.948581 (−7.6200)	−0.0160
Equity non-hedge	−0.037659 (−16.591)	0.033898 (7.6781)	0.004715 (0.0503)	−0.1161
Event-driven	−0.005919 (−6.8902)	0.017125 (10.080)	0.367516 (5.0669)	−0.0679
Fixed income (total)	−0.001160 (−1.1893)	0.007781 (3.8567)	0.382432 (1.9556)	−0.0299
Fixed income: arbitrage index	−0.007122 (−10.690)	0.005307 (4.3401)	0.963810 (5.4234)	−0.0523
Fixed income: high-yield index	0.005758 (0.9201)	0.067150 (4.5804)	−0.757067 (−4.5238)	−0.0674
Fund-of-funds index	−0.010523 (−20.21)	0.005896 (6.3992)	0.759491 (6.3642)	−0.0474
Macro Index	−0.013036 (−6.3669)	0.015060 (3.8926)	0.129333 (0.6784)	−0.0534
Market-timing index	−0.014686 (−22.815)	0.009171 (6.5608)	−0.241209 (−2.1176)	−0.0309
Merger-arbitrage index	−0.001516 (−0.9552)	0.009901 (2.8773)	0.727790 (2.7998)	−0.0606
Relative-value arbitrage index	−0.001835 (−4.3789)	0.002318 (3.6110)	1.088906 (4.7789)	−0.0258
Short-selling index	−0.073448 (−40.991)	0.035989 (10.511)	0.211926 (3.0456)	−0.1802
Statistical arbitrage index	−0.004718 (−2.9738)	0.008355 (2.4503)	−0.195574 (−0.5230)	−0.0202

the location parameter (μ) that determines the average of the extremes is estimated to be negative and statistically significant at the 1% or 5% level except for a few cases. The shape parameter (ξ), called the tail index, reflects the fatness of the distribution (ie, the weight of the tails).[16] The tail-thickness parameter (ξ) is generally estimated to be positive, and significant at the 5% level.

It is important to note that the commonly used symmetric parametric VAR models do not allow for asymmetries in calculating thresholds since the positive and negative tails of the normal distribution are identical. In other words, the standard parametric VAR approach yields almost the same thresholds for the maximal and minimal changes in risk factors. One advantage of the CF expansion is that asymmetries in the positive and negative tails of the distribution can easily be handled since it separates sudden drops from sudden jumps. This allows us to test whether the two tails are indeed symmetric.

Table 2 presents the actual and estimated 1% VAR thresholds for the normal, GED, EVT and CF expansion.[17,18] We compare the estimated values with actual, empirical 1% quantiles ($\Im_{actual}$) estimated over the same period. The results show that the actual VARs are estimated more accurately by the EVT and CF expansion than by the GED and normal distributions. The normal density, GED, CF expansion and EVT yield 1% VAR thresholds that on average underestimate the actual VAR thresholds by 37%, 32%, 10% and 6% respectively. These results show that the extreme value approach and the CF expansion, which models both skewness and kurtosis, outperform the GED and normal distributions in calculating VAR.[19] In particular, for convertible arbitrage, distressed securities, event driven and merger arbitrage strategies, VAR with normal and GED distributions are not well suited. This is because these strategies contain large amounts of firm or event-specific risk, and large losses due to these risks cannot be captured by normal market conditions. We also created mean-normal VAR and mean-CF VAR efficient frontiers. These frontiers, displayed in Figure 1, show that mean-normal VAR efficient frontiers significantly underestimate the VAR of the portfolio.

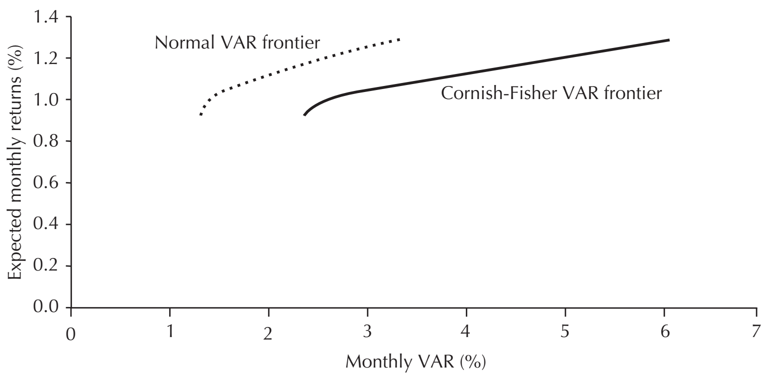
So far, VAR estimates imply that the tail areas obtained from the EVT and CF expansion are quite different from, and potentially

Table 2 Actual and estimated 1% VAR thresholds

HFRI index	$\Im_{actual}$ (%)	$\Im_{CF}$ (%)	$\Im_{EVT}$ (%)	$\Im_{GED}$ (%)	$\Im_{Normal}$ (%)	$\frac{\Im_{CF}}{\Im_{actual}}$	$\frac{\Im_{EVT}}{\Im_{actual}}$	$\frac{\Im_{GED}}{\Im_{actual}}$	$\frac{\Im_{Normal}}{\Im_{actual}}$
Convertible arbitrage	−2.99	−2.35	−2.81	−1.45	−1.31	0.78	0.94	0.48	0.44
Distressed securities	−6.04	−5.99	−5.68	−3.75	−3.09	0.99	0.94	0.62	0.51
Emerging markets (total)	−16.55	−14.44	−14.28	−10.39	−9.39	0.87	0.86	0.63	0.57
Equity hedge	−5.98	−5.12	−5.69	−5.02	−4.73	0.86	0.95	0.84	0.79
Equity-market-neutral	−1.62	−1.39	−1.60	−1.47	−1.31	0.86	0.99	0.91	0.81
Equity non-hedge	−12.00	−10.15	−11.61	−8.68	−8.50	0.85	0.97	0.72	0.71
Event-driven	−7.30	−6.14	−6.79	−3.53	−3.33	0.84	0.93	0.48	0.46
Fixed income (total)	−3.20	−2.82	−2.99	−1.85	−1.52	0.88	0.93	0.58	0.48
Fixed income: arbitrage index	−6.27	−5.29	−5.23	−2.99	−2.44	0.84	0.83	0.48	0.39
Fixed income: high-yield index	−6.67	−6.97	−6.74	−4.45	−3.80	1.05	1.01	0.67	0.57
Fund-of-funds index	−5.42	−4.94	−4.74	−3.74	−3.15	0.91	0.87	0.69	0.58
Macro index	−5.09	−4.16	−5.34	−4.85	−4.57	0.82	1.05	0.95	0.90
Market-timing index	−3.14	−3.13	−3.09	−3.11	−3.58	1.00	0.98	0.99	1.14
Merger-arbitrage index	−6.08	−4.40	−6.06	−1.85	−2.08	0.72	1.00	0.30	0.34
Relative-value arbitrage index	−3.50	−4.51	−2.58	−1.75	−1.50	1.29	0.74	0.50	0.43
Short-selling index	−18.73	−16.00	−18.02	−16.49	−14.88	0.85	0.96	0.88	0.79
Statistical arbitrage index	−2.00	−1.80	−2.02	−1.79	−1.76	0.90	1.01	0.90	0.88

This table presents the actual and estimated 1% VAR thresholds for the normal, GED, EVT and CF expansion.
The actual VAR threshold is identified by the 1% left tail of the empirical distribution. The ratios of the actual VARs to the estimated VARs are reported to compare the relative performance of alternative VAR models. The average values of the ratios are $\Im_{CF}/\Im_{actual} = 0.90$, $\Im_{EVT}/\Im_{actual} = 0.94$, $\Im_{GED}/\Im_{actual} = 0.68$, and $\Im_{Normal}/\Im_{actual} = 0.63$.

Figure 1 Mean-normal VAR and mean-CF VAR efficient frontiers



more useful than, the tails obtained with the GED and normal distributions. The implied VAR estimates are consequently different also. Yet, the real test for the EVT and CF approach is not that it is different from the alternatives, but whether it is any better according to some specific criteria. One of the most popular evaluation criteria is the regression R^2 obtained from the regression of actual VAR thresholds on estimated thresholds:

$$\Im_{\text{actual},i} = a_0 + a_1 \Im_{\text{estimated},i} + u_i \tag{13}$$

where $\Im_{\text{actual},i}$ is the actual VAR threshold for hedge fund index return i, $\Im_{\text{estimated},i}$ is the estimated threshold based on the normal, GED, EVT and CF expansion, ie, $\Im_{\text{estimated}} = \Im_{\text{Normal}}$, $\Im_{\text{GED}}$, $\Im_{\text{EVT}}$, or $\Im_{\text{CF}}$. R^2 values provide information about how well each VAR model is able to estimate the actual VAR threshold.

As an alternative to R^2 measures, we also compute the TIC, which always lies between zero and one, where zero indicates a perfect fit:

$$\text{TIC} = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Im_{\text{actual},i} - \Im_{\text{estimated},i})^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Im_{\text{actual},i})^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\Im_{\text{estimated},i})^2}} \tag{14}$$

The TIC values describe the fit of the normal, GED, EVT and CF expansion for the actual VAR of hedge fund returns.

Using the actual and estimated 1% VAR thresholds given in Table 2, we calculate the R^2 and TIC values for hedge fund index returns. The results highlight the superior performance of the CF expansion against the GED and normal distributions. Specifically, the R^2 values are found to be 98.89% for the EVT, 98.13% for the CF, 89.75% for the GED density and 88.50% for the standard approach with normal density. The TIC measures turn out to be 4.59% for the EVT, 7.82% for the CF, 18.75% for the GED and 22.53% for the normal distribution. As expected, the CF expansion, which accounts for the skewness and kurtosis of hedge fund returns, performs better than the GED and normal distributions.

Both the R^2 and TIC compute the total variation in $\Im_{\text{actual},i}$ explained by $\Im_{\text{Normal}}$, $\Im_{\text{GED}}$, $\Im_{\text{EVT}}$ or $\Im_{\text{CF}}$. Although they provide the direction and magnitude of the relationship between the actual and estimated VAR thresholds, neither R^2 nor TIC measures how far the estimated threshold is away from the actual. To compute directly the deviation between $\Im_{\text{actual},i}$ and $\Im_{\text{estimated},i}$, we use the HMAE and HRMSE:[20]

$$\text{HMAE} = \frac{1}{n}\sum_{i=1}^{n}\left|\left(1-\frac{\Im_{\text{actual},i}}{\Im_{\text{estimated},i}}\right)\right| \tag{15}$$

$$\text{HRMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{\Im_{\text{actual},i}}{\Im_{\text{estimated},i}}\right)^2} \tag{16}$$

These two forecast criteria follow the statistical tradition of reporting statistics based directly on the deviation between actual and estimated, while adjusting for heteroscedasticity in the forecast error.

Table 3 provides the HMAE and HRMSE values for alternative VAR models. The average mean absolute (percentage) error turns out to be 7.96% for the EVT, 15.91% for the CF expansion, 61.01% for the GED and 77.30% for the normal density. Similarly, the average root mean square (percentage) error is found to be 11.87% for the EVT, 18.34% for the CF expansion, 82.65% for the GED, and 94.64% for the normal distribution. These results confirm our earlier findings based on the R^2 and TIC measures, and indicate that the EVT and

Table 3 Relative performance of alternative approaches to estimating VAR

HFRI index	HMAE				HRMSE			
	CF (%)	EVT (%)	GED (%)	Normal (%)	CF (%)	EVT (%)	GED (%)	Normal (%)
Convertible arbitrage	27.42	6.41	106.21	128.23	7.52	0.41	112.80	164.43
Distressed securities	0.86	6.34	61.07	95.68	0.01	0.40	37.29	91.55
Emerging markets (total)	14.59	15.90	59.24	76.24	2.13	2.53	35.09	58.12
Equity hedge	16.62	5.10	19.02	26.41	2.76	0.26	3.62	6.97
Equity-market-neutral	16.38	1.25	10.20	23.47	2.68	0.02	1.04	5.51
Equity non-hedge	18.24	3.36	38.25	41.12	3.33	0.11	14.63	16.90
Event-driven	18.78	7.51	106.66	119.31	3.53	0.56	113.76	142.35
Fixed income (total)	13.62	7.02	72.97	110.16	1.85	0.49	53.25	121.35
Fixed income: arbitrage index	18.44	19.89	109.70	157.34	3.40	3.95	120.34	247.57
Fixed income: high-yield index	4.35	1.04	49.78	75.18	0.19	0.01	24.78	56.52
Fund-of-funds index	9.74	14.35	44.92	72.08	0.95	2.06	20.18	51.96
Macro index	22.37	4.68	4.95	11.26	5.00	0.22	0.24	1.27
Market-timing index	0.11	1.62	0.80	12.51	0.00	0.03	0.01	1.57
Merger-arbitrage index	38.15	0.33	228.38	192.64	14.55	0.00	521.57	371.09
Relative-value arbitrage index	22.58	35.66	99.71	132.97	5.10	12.72	99.43	176.80
Short-selling index	17.07	3.94	13.55	25.83	2.91	0.16	1.84	6.67
Statistical arbitrage index	11.23	0.99	11.73	13.67	1.26	0.01	1.38	1.87
Average	**15.91**	**7.96**	**61.01**	**77.30**	**18.34**	**11.87**	**82.65**	**94.64**

The HMAE and HRMSE are used to compute directly the deviation between $\Im_{actual,i}$ and $\Im_{estimated,i}$. These two forecast criteria given in equations (15)−(16) follow the statistical tradition of reporting statistics based directly on the deviation between actual and estimated, while adjusting for heteroscedasticity in the forecast error.

CF expansion performs much better than the symmetric fat-tailed GED and thin-tailed normal distributions in capturing the extent of extreme events in the hedge fund market.[21]

Conclusion

We provide an empirical study on how well VAR models capture the behaviour of the tails of the return distribution of hedge fund strategy indices, that is, those rare but important instances in which large losses are realised. This is a fundamental issue in VAR modelling, especially for hedge funds, since most hedge fund return distributions display a significant amount of tail risk. In fact, one of the key motivations of the development of VAR models was to be able to answer the question: if something goes wrong, how much money am I likely to lose? Put differently, hedge fund managers and investors in hedge funds want to be able to assess the extent of losses in the event of adverse movements in financial markets. Thus, the ability to model precisely the tails of the distribution is an important concern. This ability is especially key when the return distribution features fat tail risk as in hedge funds.

We compare the empirical performance of the thin-tailed normal, the fat-tailed GEDs, EVT and the CF expansion in computing VAR thresholds for hedge fund index returns. Alternative performance measures – R^2, TIC, HMAE and HRMSE – are used to assess the accuracy and performance of alternative VAR models. The results, based on alternative performance measures, indicate that the size of actual losses is much higher than that predicted by the GED and normal distributions. The symmetric GED density performs better than the symmetric normal distribution, but still does not allow for asymmetric treatment of upside potential and downside risk. An important advantage of the EVT and CF expansion is that it allows investors to treat losses and gains asymmetrically.

The EVT and CF methods performed best among those examined here. Estimating EVT parameters, however, is not an easy task. In contrast CF expansion is easily obtained. Although the extreme value approach provides slightly more accurate estimates of the actual VAR thresholds, practitioners may still be willing to use the CF expansion because of its simplicity.

1 For example, if the given period of time is 1 day and the given probability is 1%, the VAR measure would be an estimate of the decline in portfolio value that could occur with a 1% probability over the next trading day. That is, if the VAR measure is accurate, losses greater than the VAR measure should occur less than 1% of the time.

2 Asness, Krail and Liew (2001) find evidence that strongly suggests non-synchronous pricing problems for hedge funds whether due to stale or managed prices. They conclude that this is a significant issue and can lead to severely understated estimates of hedge fund risk. On the other hand, using reported prices also has limitations because they may not reflect market-clearing prices.

3 Hull and White (1998) and Venkataraman (1997) show that the risk-management performance of standard VAR models that assumes normality increases if one uses a mixture of normal distributions and quasi-Bayesian estimation techniques.

4 The two most popular VAR techniques are the variance-covariance analysis and historical simulation. Historical simulation does not rely on normality and so it does not suffer from the tail-bias problem. By applying the empirical distribution of all assets' returns in the trading portfolio, the outcome will reflect the historical frequency of large losses over the specified data window. Unlike the variance-covariance analysis, the historical approach can be used in a natural way to compute VAR for nonlinear positions, such as derivative positions. In addition to these two, fully non-parametric approaches have been proposed and determine the empirical quantile or a smoothed version of it (Harrel and Davis, 1982; Jorion, 1996; Gourieroux, Laurent and Scaillet, 2000). Recently, semi-parametric approaches have been developed. They are based on either extreme value distributions (Longin, 2000; McNeil and Frey, 2000; Bali, 2001) or local likelihood methods (Gourieroux and Jasiak, 1999). For a comprehensive survey on value at risk models, see Duffie and Pan (1997), Dowd (1998) and Jorion (2001).

5 The thresholds for the standard approach, $\Im_{\text{Normal}}$, are computed using the estimated mean and volatility parameters of the normal distribution as well as the critical values: 2.5758, 2.326, 2.1701, 2.0536, 1.960 and 1.645 for the 0.5%, 1%, 1.5%, 2%, 2.5% and 5% VAR tails, respectively.

6 For example, $z(\alpha)$ equals -2.326 (-1.960) [-1.645] for the 1% (2.5%) [5%] VAR.

7 The *shape* parameter ξ, called the tail index, reflects the fatness of the distribution (ie, the weight of the tails), while the parameters of *scale* σ and of *location* μ represent the dispersion and average of the extremes, respectively.

8 Details and presentation of alternative statistical estimation procedures of GPD can be found in Leadbetter, Lindgren and Rootzen (1983), Castillo (1988), Embrechts, Kluppelberg and Mikosch (1997) and Bali (2001).

9 For example, a random variable distributed as the normal gives $l = -\infty$ and $u = +\infty$.

10 Survivorship bias is caused by eliminating closed or liquidated hedge funds from an index, and keeping only live or successful hedge funds in the index. As a result returns are overestimated. It is a question of whether index returns and higher moments represent only the successful funds or are true representation of all the funds including liquidated ones.

11 It is well known that all major hedge fund data vendors started to collect the defunct hedge fund data beginning in 1994. Hence hedge fund returns prior to 1994 suffer from survivorship bias. As a result, all the statistics based on our sample period (January 1990–June 2002) would be affected by this survivorship bias. Therefore, Table 1 reports the descriptive statistics of hedge fund index returns for the sample period of January 1994 to June 2002. We should note that the estimated parameters of the normal, GED and GPD distributions do not change much in the shorter and the longer sample periods. The qualitative results from the period of January 1994–June 2002 turn out to be very similar to those reported in our tables.

12 Note that the critical values for CF expansion, $\Omega(\alpha)$, account for skewness and kurtosis of the empirical distribution.

13 The VARs are calculated on a monthly horizon because the HFR data include monthly index returns.

14 Panel 1 provides a short discussion on the estimation procedure of the GPD.

15 Note that the previous studies on EVT use the 2.5% and 5% left tail of the empirical distribution to obtain the excesses over high thresholds. However, the existing literature generally uses daily datasets with large number of observations. Since we do not have large number of observations (150 monthly observations from January 1990 to June 2002), we define the high threshold as the 10% (instead of 2.5% or 5%) left tail of the return distribution.

16 Notice that the GPD presented in equation (10) encompasses the Pareto distribution, the *uniform* distribution on $[-1, 0]$, and the standard *exponential* distribution. For $\xi > 0$, $\xi < 0$, and $\xi = 0$ we obtain the Pareto, uniform and exponential distributions, respectively. The Pareto distribution (with $\xi > 0$) is fat-tailed as its tail is slowly decreasing; the uniform distribution (with $\xi < 0$) is short-tailed – after a certain point there are no extremes; the exponential distribution (with $\xi = 0$) is thin-tailed as its tail is rapidly decreasing.

17 We also compare the relative performance of normal, GED, EVT, and CF expansion for the 2.5% and 5% VAR thresholds. The results turn out to be very similar to those reported in our tables. To save space, we do not present the empirical findings based on the 2.5% and 5% VARs. They are available upon request.

18 Note that the actual VAR threshold is identified by the 1% left tail of the empirical distribution. Although defining the actual threshold does not need a certain distribution function, it still requires that the return process to be stationary. Although not presented in the chapter, the Augmented Dickey-Fuller (ADF) (1973) statistics indicate rejection of the null hypothesis of a unit root for the hedge fund index returns at the 1% level. The full set of details on the unit root tests is available upon request.

19 To some extent, these results were expected. They reflect the more flexible specifications and the in-sample fitting. Future research should focus on out-of-sample performance.

20 The HMAE and HRMSE are used by Andersen, Bollerslev and Lange (1999).

21 To check the robustness of our results, we use a broader index of Hedge Fund Research, Inc. (HFR) and evaluate the relative performance of alternative VAR models for hedge fund portfolios. These results confirm our earlier findings that the EVT and CF expansion performs extremely well in estimating value at risk of hedge fund portfolios.

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